Exercise 8

Differentiate.

$$f(t) = \frac{\cot t}{e^t}$$

Solution

Use the product and quotient rules to differentiate $f(t)$.

$$\begin{aligned}
f'(t) &= \frac{d}{dt}[f(t)] \\
&= \frac{d}{dt}\left(\frac{1}{e^t \tan t}\right) \\
&= \frac{\left[\frac{d}{dt}(1)\right](e^t \tan t) - \left[\frac{d}{dt}(e^t \tan t)\right](1)}{(e^t \tan t)^2} \\
&= \frac{(0)(e^t \tan t) - \left\{\left[\frac{d}{dt}(e^t)\right]\tan t + e^t\left[\frac{d}{dt}(\tan t)\right]\right\}(1)}{e^{2t}\tan^2 t} \\
&= \frac{0 - \left[(e^t)\tan t + e^t(\sec^2 t)\right](1)}{e^{2t}\tan^2 t} \\
&= -\frac{(e^t)\tan t + e^t(\sec^2 t)}{e^{2t}\tan^2 t} \\
&= -\frac{\tan t + \sec^2 t}{e^t \tan^2 t} \\
&= -\frac{\frac{\sin t}{\cos t} + \frac{1}{\cos^2 t}}{e^t \frac{\sin^2 t}{\cos^2 t}} \cdot \frac{\cos^2 t}{\cos^2 t} \\
&= -\frac{\sin t \cos t + 1}{e^t \sin^2 t}
\end{aligned}$$